Hot water tank: Formulae

In this activity you will learn

- how to substitute numbers into formulae
- how to use a calculator to evaluate complicated expressions.

Information sheet

You can find how much water a tank can hold by using the **formula**

$V = \pi r^2 h$

Think about ...

What do the letters represent in this formula?

Formulae

Fixed values such as the number of days in a week (7) are called **constants**.
π is a constant whose value is approximately 3.14; a more accurate value is programmed into scientific calculators.

Values that are not fixed are called **variables**.
Letters can be used to represent variables.
Relationships between the variables are summarised using **formulae**.

Substituting particular values into a formula gives information about particular cases.

For example, the volume of this tank

$V = \pi r^2 h = \pi \times 30^2 \times 150$

$= \pi \times 30 \times 30 \times 150 = 424\ 115\ \text{cm}^3$

Think about ...

Is this a sensible way to give the answer?

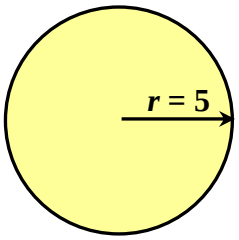
1000 cm^3 = 1 litre, so the volume is 424 litres (to the nearest litre)

Think about ...

Is this a reasonable answer?
How many baths is it? (A bath holds about 80 litres.)

Example: area of a circle

The area of a circle of radius r is given by the formula $A = \pi r^2$



Find the area of a circle with radius 5 cm.

How to do it ...

Substituting $r = 5$ into the formula for the area of a circle gives
$A = \pi r^2 = \pi \times 5^2 = \pi \times 5 \times 5 = 78.539 \ldots$

Think about ...

How far should this be rounded?

The area of the circle whose radius is 5 cm is **79 cm^2 (to the nearest cm^2).**

Example: calculating interest

When £P is left in a building society account giving r % interest per annum, the amount in the account after n years is given by $A = P\left(1 + \frac{r}{100}\right)^n$.

If £750 is invested in an account giving 4.5% per annum interest, find the amount in the account after 6 years.

How to do it ...

Substitute $P = 750$, $r = 4.5$ and $n = 6$ to give $A = 750\left(1 + \frac{4.5}{100}\right)^6$

Think about...

How do you work this out? $A = 750 \times 1.045^6 = 976.695\ldots$.

The amount in the account after 6 years is **£976.70 (to the nearest pence).**

Example: radius of a sphere from its volume

The radius of a sphere is $r = \sqrt[3]{\frac{3V}{4\pi}}$ where V is its volume.

Find the radius of a ball bearing whose volume is 9.6 mm^3.

How to do it.....

Substitute $V = 9.6$ into $r = \sqrt[3]{\frac{3V}{4\pi}}$, giving $r = \sqrt[3]{\frac{(3 \times 9.6)}{(4 \times \pi)}}$

Think about...

How do you work this out on a calculator?

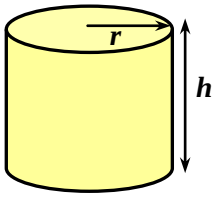
$$r = \sqrt[3]{2.29183..} = 1.318\ldots$$

The radius of the ball bearing is **1.3 mm (to 2 sf)**

Think about...

Can you think of other examples where you have met formulae before?

Try these



1 The volume of a cylinder is given by the formula $V = \pi r^2 h$

where r is the radius and h is the height.

Find the volume of a cylindrical can with radius 4 cm and height 7.5 cm.

2 The speed of a car after t seconds is given by the formula $v = u + at$
where u is the starting speed in metres per second (m s^{-1}) and
a is the acceleration in metres per second per second (m s^{-2}).

A car joins a motorway at a speed of 25 m s^{-1} and accelerates at 0.65 m s^{-2} for 12 seconds.

Find its speed at the end of this time.

3 If a ball is thrown up in the air at a velocity of v m s^{-1}, the height it reaches is given by $h = \dfrac{v^2}{2g}$, where g is the acceleration due to gravity in m s^{-2}.

(Note that g is approximately 9.81 m s^{-2})

Find the height reached by a ball thrown up at a velocity of 15 m s^{-1}.

4 The formula for converting temperatures is $C = \dfrac{5(F - 32)}{9}$
where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

An old recipe gives the oven temperature 375 °F. Convert this temperature to Celsius.

5 When an object is released so that it falls to the ground,
it reaches a velocity given by $v = \sqrt{2gh}$ where h is the height (in metres)
from which it is dropped and g is approximately 9.81 m s^{-2}.

If a stone is dropped from a bridge at a height of 50 metres above a river,
find its velocity on impact.

6 When a car accelerates from a speed of u m s^{-1} to a speed of v m s^{-1}
in t seconds, the distance it travels (in metres) is given by the formula: $d = \dfrac{t(u + v)}{2}$

Find the distance travelled by a car that accelerates from 17.5 m s^{-1} to 32 m s^{-1} in 9.5 seconds.

7 The time taken for a pendulum to make one full swing is $T = 2\pi\sqrt{\frac{l}{g}}$

where l metres is the length of the pendulum and g is approximately 9.81.

Find the time it takes a pendulum of length 0.5 metres to make a full swing.

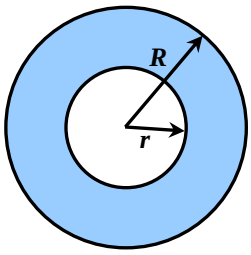
8 If the monthly rate of interest on a loan is m %, the annual percentage rate (APR) is given by the formula below.

$$\text{APR} = 100\left[\left(1+\frac{m}{100}\right)^{12} - 1\right]$$

Find the APR if the monthly rate is 1.25%.

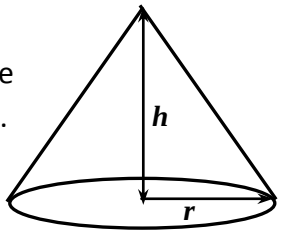
9 The area of a washer is given by the formula $A = \pi\left(R^2 - r^2\right)$ where r is the inner radius and R is the outer radius.

Find the area when r = 5 mm and R = 7.5 mm.



10 The formula $A = \pi r\sqrt{r^2 + h^2}$ gives the area of the curved surface of a cone with radius r and height h.

Find A when r = 7.5 cm and h = 8.2 cm.



Reflect on your work

- The formula for the volume of a tank is $V = \pi r^2 h$
  Which of the letters are variables and which is a constant?
- How do you decide in what order to press the calculator buttons?
- Were there any examples where you found it difficult to use the calculator correctly?
- Which did you find the most complicated?